Goniometria

Esprimere in radianti i seguenti angoli (o archi):

1. a) 14° b) 125° c) 126° d) 252° e) 486°

2. a) 30° 45' b) 192° 30' c) 252° 24' d) 227° 12' e) 602° 24'

3. a) 59° 3' 45" b) 150° 37' 30" c) 261° 15' 18" d) 297° 26' 15" e) 286° 52' 30"

Esprimere in gradi sessagesimali i seguenti angoli (o archi):

4. a) $\frac{7}{45}\pi$ b) $\frac{17}{10}\pi$ c) $\frac{13}{18}\pi$ d) $\frac{19}{12}\pi$ e) $\frac{25}{36}\pi$

5. a) $\frac{13}{75}\pi$ b) $\frac{29}{50}\pi$ c) $\frac{5}{8}\pi$ d) $\frac{31}{16}\pi$ e) $\frac{23}{8}\pi$

6. a) $\frac{13}{64}\pi$ b) $\frac{37}{125}\pi$ c) $\frac{51}{32}\pi$ d) $\frac{57}{32}\pi$ e) $\frac{253}{125}\pi$

Calcolare il valore delle seguenti espressioni:

14. a) $\sin\frac{\pi}{2} - \cot\frac{\pi}{4} + \sin\pi + 2\cos\frac{\pi}{6}$ b) $2\sin\frac{\pi}{3} - \cos 0 + 3\cos\pi - \cot\frac{\pi}{6}$

15. a) $\sin\frac{\pi}{2} - \sin\frac{3}{2}\pi - \tan\frac{\pi}{6} + \frac{1}{3}\cot\frac{\pi}{6}$ b) $\cos\frac{\pi}{3} - 3\sin\pi + 4\sin\frac{\pi}{4} - 5\cos\frac{\pi}{4} - \sin\frac{\pi}{6}$

16. a) $\dfrac{\cos\frac{\pi}{2}\sin\frac{3}{2}\pi - 2\sin^2\frac{\pi}{2}\cos\frac{3}{2}\pi}{\cos^2\frac{3}{2}\pi + 2\cos\frac{\pi}{6}}$ b) $\dfrac{\sin 30° - 2\cos 60°\cot 30° + \tan 60° - \cot 45°}{3\tan 30° - 2\sin 60° - \cot 30°}$

17. $\dfrac{\left(a\cos 0 + b\sin\frac{\pi}{2}\right)\left(a\sin\frac{\pi}{2} + b\cos\pi\right) + 2ab\sin\frac{3}{2}\pi + 2b^2\cos 2\pi}{-a\cos\pi - b\tan\frac{\pi}{4}}$, $a \neq b$

18. $\dfrac{4(a-1)^2\sin\frac{\pi}{6} - 2a\cos\pi + \tan^2\frac{\pi}{3} + \sin\frac{3}{2}\pi\left(a^2 + 2a\tan\frac{\pi}{4} + 4\cos^2\frac{\pi}{3}\right)}{a\cos 2\pi - 4\sin^2\frac{\pi}{4} + \tan^2\frac{\pi}{3} - \cot^2\frac{\pi}{6}}$, $a \neq 2$

19. a) $\dfrac{2ab}{\tan\frac{\pi}{4}} - b^2 - (a-b)^2\cos\pi - a^2\cot\frac{\pi}{4}$

b) $\left(a\cos\frac{\pi}{2} + b\tan\frac{\pi}{4} - \cot\frac{\pi}{4}\right)\left(a\tan 2\pi - b\sin\frac{3}{2}\pi + \tan\frac{\pi}{4}\right)$

20. a) $\sin 10\pi - \cos 11\pi + \sin 12\pi - \cos 18\pi$ b) $2\sin\frac{5}{2}\pi - 2\tan\frac{13}{4}\pi - \left(\cos\frac{7}{4}\pi - \sin\frac{3}{4}\pi\right)^2$

21. a) $\dfrac{\tan\frac{13}{4}\pi\left(\cot\frac{15}{4}\pi+\cos 5\pi\right)}{\sin\frac{7}{2}\pi\left(\sin\frac{5}{2}\pi-\cos 6\pi\right)+2\tan\frac{5}{4}\pi}$ b) $\dfrac{\frac{1}{4}\left(2\sin 4\pi-3\sin\frac{7}{2}\pi\right)}{\sin 3\pi-\cos\frac{\pi}{3}\cos 5\pi}:\dfrac{2\cos 6\pi+\tan\frac{5}{4}\pi}{2(\sin 3\pi-\cos 3\pi)}$

22. a) $\left(a\cos^3 3\pi+b\sin^2\frac{3}{2}\pi\right)^2-\left(a\cos^2 4\pi-b\sin\frac{7}{2}\pi\right)^2$

b) $\left(\sin 4\pi+a\sin\frac{3}{2}\pi\right)^2-(\sin 7\pi-a\cos 5\pi)^2+4ab\sin\frac{\pi}{6}$

23. a) $\dfrac{a(a\cos 20\pi+b\sin 21\pi)-b(a\sin 15\pi+b\cos 16\pi)}{a(\tan 19\pi-\cos 19\pi)-b(\tan 6\pi+\cos 6\pi)}$

Tenendo presente la periodicità delle funzioni goniometriche, verificare che:

24. a) $\sin 1890°=1$ b) $\sin 1350°=-1$ c) $\sin 4050°=1$

25. a) $\sin 3285°=\frac{\sqrt{2}}{2}$ b) $\sin 1470°=\frac{1}{2}$ c) $\sin 1860°=\frac{\sqrt{3}}{2}$

26. a) $\sin\frac{37}{3}\pi=\frac{\sqrt{3}}{2}$ b) $\sin\frac{22}{5}\pi=\sin\frac{2}{5}\pi$ c) $\sin\frac{67}{8}\pi=\sin\frac{3}{8}\pi$

27. a) $\sin\frac{41}{4}\pi=\frac{\sqrt{2}}{2}$ b) $\sin\frac{86}{7}\pi=\sin\frac{2}{7}\pi$ c) $\sin\frac{37}{6}\pi=\frac{1}{2}$

28. a) $\cos 1530°=0$ b) $\cos 1980°=-1$ c) $\cos 7110°=0$

29. a) $\cos 2910°=\frac{\sqrt{3}}{2}$ b) $\cos 1845°=\frac{\sqrt{2}}{2}$ c) $\cos 3300°=\frac{1}{2}$

30. a) $\cos\frac{19}{3}\pi=\frac{1}{2}$ b) $\cos\frac{53}{5}\pi=\cos\frac{3}{5}$ c) $\cos\frac{69}{8}\pi=\cos\frac{5}{8}\pi$

31. a) $\cos\frac{41}{6}\pi=\cos\frac{5}{6}\pi$ b) $\cos\frac{35}{4}\pi=-\frac{\sqrt{2}}{2}$ c) $\cos\frac{75}{7}\pi=\cos\frac{5}{7}\pi$

32. a) $\tan 3780°=0$ b) $\tan 930°=\frac{\sqrt{3}}{3}$ c) $\tan 2025°=1$

33. a) $\tan 1680°=\sqrt{3}$ b) $\tan\frac{19}{6}\pi=\frac{\sqrt{3}}{3}$ c) $\tan\frac{38}{7}\pi=\tan\frac{3}{7}\pi$

34. a) $\tan\frac{27}{4}\pi=-1$ b) $\tan\frac{20}{3}\pi=\tan\frac{2}{3}\pi$ c) $\tan\frac{71}{8}\pi=\tan\frac{7}{8}\pi$

35. a) $\cot 855°=-1$ b) $\cot 1350°=0$ c) $\cot 1110°=\sqrt{3}$

36. a) $\cot 2220° = \frac{\sqrt{3}}{3}$ b) $\cot \frac{49}{6}\pi = \sqrt{3}$ c) $\cot \frac{73}{7}\pi = \cot \frac{3}{7}\pi$

37. a) $\cot \frac{39}{4}\pi = -1$ b) $\cot \frac{22}{3}\pi = \frac{\sqrt{3}}{3}$ c) $\cot \frac{45}{8}\pi = \cot \frac{5}{8}\pi$

Dato il valore di una funzione goniometrica dell'angolo α determinare il valore delle rimanenti funzioni nei casi a fianco indicati:

46. a) $\sin\alpha = \frac{1}{3}$, $\alpha \in$ I quadrante b) $\sin\alpha = -\frac{3}{5}$, $\alpha \in$ IV quadrante

47. a) $\sin\alpha = \frac{\sqrt{2}}{3}$, $\alpha \in$ II quadrante b) $\sin\alpha = -\frac{2}{3}$, $\alpha \in$ III quadrante

48. a) $\cos\alpha = \frac{1}{4}$, $\alpha \in$ I quadrante b) $\cos\alpha = -\frac{1}{2}$, $\alpha \in$ II quadrante

49. a) $\cos\alpha = \frac{2\sqrt{2}}{3}$, $\alpha \in$ IV quadrante b) $\cos\alpha = -\frac{\sqrt{5}}{3}$, $\alpha \in$ III quadrante

50. a) $\tan\alpha = 2$, $\alpha \in$ I quadrante b) $\tan\alpha = -\frac{3}{4}$, $\alpha \in$ IV quadrante

51. a) $\tan\alpha = \frac{\sqrt{2}}{2}$, $\alpha \in$ III quadrante b) $\tan\alpha = -\frac{1}{3}$, $\alpha \in$ II quadrante

52. a) $\cot\alpha = 2\sqrt{2}$, $\alpha \in$ I quadrante b) $\cot\alpha = -\frac{\sqrt{3}}{3}$, $\alpha \in$ II quadrante

53. a) $\cot\alpha = \frac{\sqrt{5}}{2}$, $\alpha \in$ III quadrante b) $\cot\alpha = -\frac{2}{5}$, $\alpha \in$ IV quadrante

Esprimere in funzione di $\sin\alpha$ e semplificare le seguenti espressioni:

64. a) $\dfrac{1}{1+\tan^2\alpha}-\cos^2\alpha-\cot^2\alpha-1$

b) $\tan\alpha-\dfrac{\sec\alpha}{\operatorname{cosec}\alpha}+\sin^2\alpha\operatorname{cosec}\alpha$

65. a) $\cos\alpha+\dfrac{1}{\sec\alpha}\left(\dfrac{1}{\operatorname{cosec}\alpha}-1\right)-\tan\alpha\cdot\cos^2\alpha+\sin\alpha$

b) $\dfrac{\tan^2\alpha}{\sin^2\alpha}-\dfrac{1}{\operatorname{cosec}^2\alpha}+\cos^2\alpha\tan^2\alpha$

66. a) $\tan\alpha-\dfrac{\sin\alpha\cos\alpha}{\cos^2\alpha}+\dfrac{1}{\sin\alpha}\left(1-\cos^2\alpha+\dfrac{1}{1+\tan^2\alpha}\right)$

b) $\dfrac{1}{\cot^2\alpha}-\dfrac{1}{\cos^2\alpha}\left(\dfrac{1}{\operatorname{cosec}^2\alpha}-\dfrac{2}{\sec^2\alpha}\right)$

67. a) $\dfrac{1}{\operatorname{cosec}\alpha\cdot\sin\alpha\cdot\sec\alpha}-\cos\alpha+\left[1+\dfrac{1}{\cos^2\alpha}\left(1+\sin^2\alpha\right)\right]\dfrac{\operatorname{cosec}\alpha}{2\sec^2\alpha}$

b) $\dfrac{\cos\alpha}{\cot^2\alpha\sec\alpha}-\dfrac{\cos^2\alpha\tan^2\alpha}{\sin\alpha}+\sin\alpha-1+\dfrac{\sec^2\alpha}{\tan^2\alpha\cdot\operatorname{cosec}^2\alpha}$

68. a) $\tan\alpha\left(1-\cos^2\alpha\right)\cos\alpha+\dfrac{\sin\alpha}{\sec^2\alpha}-\dfrac{\sin\alpha\cos\alpha}{\sec\alpha}-\sin\alpha$

b) $\sec^2\alpha-\dfrac{1}{\cot^2\alpha}+\dfrac{1}{\sec^2\alpha}-\dfrac{1-\cos^2\alpha}{\tan^2\alpha}-\cos^2\alpha$

Esprimere in funzione di $\cos\alpha$ e semplificare le seguenti espressioni:

69. a) $\dfrac{1}{\sec\alpha}-\dfrac{\tan^2\alpha}{1+\tan^2\alpha}+1-\dfrac{1}{1+\tan^2\alpha}$

b) $\dfrac{1}{\cos\alpha}-\sin^2\alpha\sec^2\alpha+\dfrac{1-\cos^2\alpha}{1-\sin^2\alpha}$

70. a) $\dfrac{1}{\sec^2\alpha}-\dfrac{\sec^2\alpha}{\operatorname{cosec}^2\alpha}\cdot\cos\alpha+\dfrac{2}{\operatorname{cosec}^2\alpha}+\dfrac{\tan\alpha}{\cot\alpha}\cdot\cos\alpha$ b) $\sin\alpha+\dfrac{\sec\alpha}{1+\tan^2\alpha}-\dfrac{\cos\alpha}{\cot\alpha}$

71. a) $2-\dfrac{\cos\alpha}{\operatorname{cosec}\alpha\tan\alpha\sec^2\alpha}-\sin^2\alpha\cos^2\alpha$

b) $\tan\alpha-\left(1+\tan^2\alpha\right)\sin^2\alpha\cot\alpha+\cos\alpha$

72. a) $\dfrac{\cos\alpha}{\operatorname{cosec}\alpha}\left(\dfrac{1}{\sin\alpha\cos\alpha}-\dfrac{\sin^2\alpha}{\cot\alpha}+\dfrac{\cos^2\alpha}{\tan\alpha}\right)$ b) $\dfrac{\tan\alpha}{\operatorname{cosec}\alpha}+1+\dfrac{1}{\tan^2\alpha}-\operatorname{cosec}^2\alpha+\cos\alpha$

73. a) $\dfrac{\sin\alpha}{\sec\alpha}-\dfrac{\sin^2\alpha}{\tan\alpha}+\dfrac{1}{\sec\alpha}\left(1+\dfrac{1}{\cot^2\alpha}\right)$ b) $\dfrac{1}{\cot^2\alpha}+1+\sin^2\alpha\cos\alpha-\dfrac{\sec\alpha}{\cos\alpha}$

Esprimere in funzione di $\tan\alpha$ *e semplificare le seguenti espressioni:*

74. a) $\dfrac{\operatorname{cosec}\alpha\sin^2\alpha}{\cos\alpha}-1+\cos^2\alpha+\dfrac{1-\sin^2\alpha-\left(1-\sin^2\alpha\right)^2}{\cos^2\alpha}$

b) $\dfrac{\sec\alpha}{\operatorname{cosec}\alpha}+\operatorname{cosec}\alpha\sin\alpha\left(1-\sin^2\alpha\right)-\cos^2\alpha$

75. a) $\sec\alpha-\dfrac{1}{\sec\alpha}-\dfrac{\tan\alpha}{\operatorname{cosec}\alpha}+\dfrac{\sin^2\alpha}{\cos^2\alpha}$ b) $2-\dfrac{1}{\tan^2\alpha}+\dfrac{1-2\sin^2\alpha}{\sin^2\alpha\left(1-\sin^2\alpha\right)}\cdot\dfrac{\cot^2\alpha-1}{\cot^2\alpha+1}$

76. a) $\cos^2\alpha-\sin\alpha\tan\alpha+\dfrac{1}{\cos\alpha}-\dfrac{1}{\sec\alpha}$

b) $\dfrac{1}{\sin\alpha}\left[1+\dfrac{1}{\sec\alpha}-2\left(1-\sin^2\alpha\right)+\dfrac{1}{\sec^4\alpha}\right]-\dfrac{\sin^2\alpha}{\operatorname{cosec}\alpha}$

77. a) $\operatorname{cosec}\alpha\left(\dfrac{\sin\alpha}{\sec^2\alpha}-\dfrac{1}{\operatorname{cosec}\alpha}\right)+\dfrac{\sec^2\alpha-1}{\sec^2\alpha}+\dfrac{1}{\tan\alpha}$

b) $\left(\dfrac{1}{\cos^2\alpha}-1\right)\dfrac{1}{\cos^2\alpha}+\dfrac{\tan\alpha\cot\alpha\sin^2\alpha}{\sin^2\alpha-1}$

78. a) $\dfrac{1}{\sin^2\alpha}-1+\dfrac{\sin\alpha}{\cos\alpha\tan\alpha}-\cot^2\alpha$ b) $\dfrac{\tan\alpha}{1+\tan^2\alpha}+\sin\alpha\sec\alpha+\dfrac{\sin^3\alpha}{\cos\alpha}-\dfrac{1}{\cot\alpha}$

Esprimere in funzione di $\cot\alpha$ *e semplificare le seguenti espressioni:*

79. a) $\dfrac{\sec\alpha-\operatorname{cosec}\alpha}{\sec\alpha+\operatorname{cosec}\alpha}+\dfrac{\sec\alpha+\operatorname{cosec}\alpha}{\sec\alpha-\operatorname{cosec}\alpha}$ b) $\dfrac{1}{\sec\alpha\sin\alpha}+\dfrac{\left(\sin^2\alpha-\cos^2\alpha\right)\sec\alpha}{\sin\alpha}$

80. a) $\dfrac{1}{\sin^2\alpha}+\dfrac{1}{\cos^2\alpha}-\sec^2\alpha\cot^2\alpha-1$ b) $\dfrac{1}{\cos^2\alpha}-\dfrac{1}{\cot^2\alpha}+\operatorname{cosec}^2\alpha-\cos^2\alpha\operatorname{cosec}^2\alpha$

81. a) $\dfrac{2\cos^2\alpha-1}{1-\cos^2\alpha}\cdot\dfrac{\cot\alpha+\tan\alpha}{\cot\alpha-\tan\alpha}$ b) $\dfrac{\sin\alpha-1}{\cos\alpha}+\left(1-\sin^2\alpha\right)\sec^3\alpha$

82. a) $\dfrac{\sec\alpha}{\operatorname{cosec}\alpha}+\dfrac{\cos^2\alpha-\sin^2\alpha}{\sin\alpha\cos\alpha}$ b) $\dfrac{1}{\cos^2\alpha}-\dfrac{1}{\cot^2\alpha}+\operatorname{cosec}^2\alpha-\dfrac{1-\sin^2\alpha}{\sin^2\alpha}$

83. a) $\dfrac{\cot^2\alpha+\operatorname{cosec}^2\alpha}{\operatorname{cosec}^2\alpha\cot^2\alpha}\cdot\dfrac{\sin^2\alpha\tan^2\alpha}{\sin^2\alpha+\tan^2\alpha}-\dfrac{\sin^2\alpha}{\operatorname{cosec}^2\alpha\cos^2\alpha}$

b) $\dfrac{\left(\cos^2\alpha-\sin\alpha\right)\sec\alpha}{\sin\alpha}+\dfrac{1}{\sec\alpha\cos^2\alpha}$

Dimostrare le seguenti identità:

121. a) $\sqrt{\frac{1}{\cos^2\alpha}-\frac{2}{\cot\alpha}}=\tan\alpha-1$ b) $1+\frac{1}{\cot^2\alpha}+\frac{2}{\cot\alpha}+\left(1-\frac{1}{\cot^2\alpha}\right)^2=2\sec^2\alpha$

122. a) $2(1+\cos\alpha)(1+\sin\alpha)=\left(1+\frac{1}{\sec\alpha}+\frac{1}{\csc\alpha}\right)^2$ b) $\cos^4\alpha=\sin^4\alpha+1-2\sin^2\alpha$

123. a) $\frac{1}{\csc\alpha}+\tan\alpha=\frac{(1+\sin\alpha)(1+\cos\alpha)\sec\alpha\tan\alpha}{\sec\alpha+\tan\alpha}$

b) $\left(1-\frac{1}{\cot^2\alpha}\right)\cos^2\alpha=2\cos^2\alpha-1$

124. a) $\cos^4\alpha-\sin^4\alpha=\cos^2\alpha-\sin^2\alpha$ b) $\frac{1}{\cot^2\alpha}-\frac{1-3\cos^2\alpha}{\cot\cos^2\alpha}=2$

125. a) $\frac{\cot\alpha}{\cot^2\alpha-1}=\frac{\tan\alpha}{1-\tan^2\alpha}$ b) $\frac{1}{\tan\alpha}+\frac{1}{\cot\alpha}=\frac{\sec\alpha+\csc\alpha}{\sin\alpha+\cos\alpha}$

126. a) $\frac{\csc^2\alpha+\cot^2\alpha}{\sin^2\alpha+\tan^2\alpha}=\frac{\csc^2\alpha}{\tan^2\alpha}$ b) $\frac{\sin^3\alpha-\cos^3\alpha}{\sin\alpha-\cos\alpha}=\frac{1+\sec\alpha\csc\alpha}{\sec\alpha\csc\alpha}$

127. a) $\frac{\sin^3\alpha+\cos^3\alpha}{\sin\alpha+\cos\alpha}=\frac{\sec\alpha\csc\alpha-1}{\sec\alpha\csc\alpha}$ b) $(\tan\alpha+\cot\alpha)^2-(\tan\alpha-\cot\alpha)^2=4$

128. a) $\frac{1}{\cot\alpha}\left(\frac{1}{\sec\alpha}-\cos^3\alpha\right)=\frac{1}{\csc^3\alpha}$ b) $\frac{1}{\tan\alpha}\left(\frac{1}{\csc\alpha}-\sin^3\alpha\right)=\frac{1}{\sec^3\alpha}$

129. a) $\frac{\cot^2\alpha-1}{\cot^2\alpha}=\frac{\cos^2\alpha-\sin^2\alpha}{1-\sin^2\alpha}$ b) $\frac{1}{\tan^2\alpha-\sin^2\alpha}=\cot^2\alpha\csc^2\alpha$

130. a) $\tan^2\alpha\sec^2\alpha=\frac{1}{\cot^2\alpha-\cos^2\alpha}$ b) $\frac{\tan^2\alpha-\sin^2\alpha}{1-\cos^2\alpha}=\tan^2\alpha$

131. a) $\frac{\tan\alpha+\cot\alpha}{\tan\alpha-\cot\alpha}=\frac{1}{1-2\cos^2\alpha}$ b) $\frac{\tan\alpha-\cot\alpha}{\tan\alpha+\cot\alpha}=\frac{\tan^2\alpha-1}{\tan^2\alpha+1}$

132. a) $\frac{\cos\alpha}{1-\sin\alpha}+\frac{1+\sin\alpha}{\cos\alpha}=\frac{2\cos\alpha}{1-\sin\alpha}$ b) $\frac{\sin\alpha}{1+\cos\alpha}+\frac{1-\cos\alpha}{\sin\alpha}=\frac{2\sin\alpha}{1+\cos\alpha}$

133. a) $\frac{1-\sin\alpha}{1+\sin\alpha}-\tan^2\alpha=\frac{2\sin\alpha-1}{\sin^2\alpha-1}$ b) $\frac{1}{\csc\alpha}-\frac{1}{\cot\alpha}=\frac{\sin\alpha(\cos\alpha-1)}{\cos\alpha}$

134. a) $\sin\alpha-\frac{\tan^2\alpha}{\tan^2\alpha+1}=\frac{1}{\csc\alpha}-\sin^2\alpha$ b) $\tan\alpha-\frac{\sec\alpha}{\csc\alpha}-\sin\alpha=\frac{\cos^2\alpha-1}{\sin\alpha}$

135. a) $\frac{1}{\csc\alpha}+\frac{\cot\alpha}{\cot^2\alpha+1}-\cos\alpha=\sin\alpha(\cos\alpha+1)-\cos\alpha$

b) $\left(1-\frac{1}{\sec\alpha}\right)\frac{1}{\tan\alpha}+\frac{\cos\alpha-1}{\sin\alpha\cos\alpha}=(\cos\alpha-1)\tan\alpha$