

Prova di Matematica : Frazioni algebriche

Alunno: _____ Classe: **2F** L. Scienze Umane 21 ottobre 2023

A. Scomponi in fattori i seguenti polinomi:

$$3x^3y - 9x^2y^2 + 6x^2y$$

$$ax - ay - 2bx + 2by$$

$$x^6 - 6x^4y + 12x^2y^2 - 8y^3$$

$$9a^2 - 24ab + 16b^2$$

$$49a^2 - 36$$

$$a^2 - 2a - 15$$

B. Semplifica le seguenti frazioni algebriche:

$$\frac{10x^3y^5}{8x^4y^3}$$

$$\frac{4x^2 - 4x + 1}{2x^2 - x}$$

$$\frac{x^2 + 2x - xy - 2y}{x^2 - x - 6}$$

C. Semplifica le seguenti espressioni contenenti frazioni algebriche:

$$\frac{4x^3}{15y^4} \cdot \frac{9y^5}{2x^4y^3}$$

$$\frac{5x^4}{4x - 4} \cdot \frac{3x - 3}{x^3}$$

$$\frac{4x^2 - 1}{2x - 6} \cdot \frac{x^2 - 3x}{4x^2 - 4x + 1}$$

$$\frac{2}{3a^2b} + \frac{3}{2ab^2} - \frac{1}{a^2b^2}$$

$$\frac{y}{x^2 - xy} - \frac{x}{xy - y^2} + \frac{x - y}{xy}$$

Soluzione

A. Scomponi in fattori i seguenti polinomi:

$$1. 3x^3y - 9x^2y^2 + 6x^2y = 3x^2y \cdot (x - 3y + 2)$$

$$2. a^2 - 2a - 15 = (a + 3)(a - 5)$$

$p = -15$		$s = -2$
+3	-5	-2

$$4. ax - ay - 2bx + 2by = a(x - y) - 2b(x - y) = (x - y)(a - 2b).$$

$$5. 9a^2 - 24ab + 16b^2 = (3a - 4b)^2$$

$$6. 49a^2 - 36 = (7a + 6) \cdot (7a - 6)$$

$$7. x^6 - 6x^4y + 12x^2y^2 - 8y^3 = (x^2 - 2y)^3$$

B. Semplifica le seguenti frazioni algebriche:

$$\frac{10x^3y^5}{8x^4y^3} = \frac{5y^2}{4x}$$

$$C.E.: x \neq 0 \wedge y \neq 0$$

$$\begin{aligned} \frac{4x^2 - 4x + 1}{2x^2 - x} &= \\ &= \frac{(2x - 1)^2}{x(2x - 1)} = \frac{2x - 1}{x}. \end{aligned}$$

$$C.E.: x \neq 0 \wedge x \neq \frac{1}{2}$$

$$\begin{aligned} \frac{x^2 + 2x - xy - 2y}{x^2 - x - 6} &= \\ &= \frac{x(x + 2) - y(x + 2)}{(x + 2)(x - 3)} = \frac{(x + 2)(x - y)}{(x + 2)(x - 3)} = \frac{x - y}{x - 3} \end{aligned}$$

$$C.E.: x \neq -2 \wedge x \neq 3$$

C. Semplifica le seguenti espressioni contenenti frazioni algebriche:

$$\frac{4x^3}{15y^4} \cdot \frac{9y^5}{2x^4y^3} = \frac{6}{5xy^2}$$

$$C.E.: x \neq 0 \wedge y \neq 0$$

$$\frac{5x^4}{4x - 4} \cdot \frac{3x - 3}{x^3} = \frac{5x^4}{4(x - 1)} \cdot \frac{3(x - 1)}{x^3} = \frac{15x}{4}.$$

$$C.E.: x \neq 0 \wedge x \neq 1$$

$$\begin{aligned} \frac{4x^2 - 1}{2x - 6} \cdot \frac{x^2 - 3x}{4x^2 - 4x + 1} &= \\ &= \frac{(2x + 1)(2x - 1)}{2(x - 3)} \cdot \frac{x(x - 3)}{(2x - 1)^2} = \frac{x(2x + 1)}{2(2x - 1)}. \end{aligned}$$

$$C.E.: x \neq \frac{1}{2} \wedge x \neq 3$$

$$\begin{aligned} \frac{2}{3a^2b} + \frac{3}{2ab^2} - \frac{1}{a^2b^2} &= \\ &= \frac{4b + 9a - 6}{6a^2b^2}. \end{aligned}$$

$$C.E.: a \neq 0 \wedge b \neq 0$$

$$\begin{aligned} \frac{y}{x^2 - xy} - \frac{x}{xy - y^2} + \frac{x - y}{xy} &= \\ \frac{y}{x(x - y)} - \frac{x}{y(x - y)} + \frac{x - y}{xy} &= \\ \frac{y^2 - x^2 + (x - y)^2}{xy(x - y)} &= \\ \frac{y^2 - x^2 + x^2 + y^2 - 2xy}{xy(x - y)} &= \\ \frac{2y^2 - 2xy}{xy(x - y)} &= \\ \frac{-2y(x - y)}{xy(x - y)} &= -\frac{2}{x}. \end{aligned}$$

$$C.E.: x \neq 0 \wedge y \neq 0 \wedge x \neq y$$